Tutorial 7

Resilient Communication Systems
Prof. Dr.-Ing. Vahid Jamali
Tutor: Mohamadreza Delbari
Email: mohamadreza.delbari@rcs.tu-darmstadt.de

TECHNISCHE UNIVERSITÄT DARMSTADT

Released: November 24, 2025.

Channel estimation

RESILIENT COMMUNICATION SYSTEMS

Problem 1

Optimal training symbol design: Consider a MIMO communication system with an AWGN channel, $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}$.

- Which one of the following options is better for training symbols where $P_t \leq 2$ mW is the total power constraint?

$$\mathbf{X}_1 = \sqrt{\frac{1}{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}, \mathbf{X}_2 = \sqrt{\frac{1}{4}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}, \mathbf{X}_3 = \sqrt{\frac{2}{2}}\begin{bmatrix}0.99 & 0.2i\\ 0 & 0.99\end{bmatrix}, \mathbf{X}_4 = \sqrt{\frac{2}{2}}\begin{bmatrix}0.1 & i\\ 0 & 0.99\end{bmatrix},$$

- Now assume that

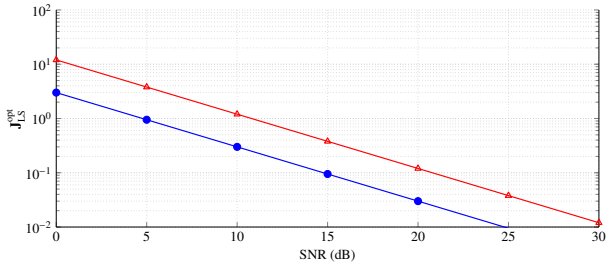
$$\mathbf{Y}_1 = \begin{bmatrix}0.1 - 0.1i & -0.2\\ 0.5i & 0.2i\\ 0.7i & -0.3\end{bmatrix},$$

  where $\mathbf{X}_1$ has been sent. Calculate the elements of the estimated channel matrix $\hat{\mathbf{H}}_{\text{LS}}$ using LS estimator.

- Calculate the total average estimation error ($\mathbf{J}_{\text{LS}}^{\text{opt}}$) for an instance when actual channel matrix is

$$\mathbf{H} = \begin{bmatrix}0.1 + 0.1i & -0.3 - 0.2i\\ 0.2 + 0.5i & 0.4 + 0.1i\\ 1i & 0.4\end{bmatrix}.$$

- Based on the following figure, determine the number of transmitter and receiver antennas. ($M_{\text{red line}} = 2M_{\text{blue line}}$ and $N_{\text{red line}} = N_{\text{blue line}}$)



Problem 2

Further reduction of the channel estimation error!
Assume $\alpha(\mathbf{H}) = \text{tr}\{\mathbb{E}\{\mathbf{H}^{\text{H}}\mathbf{H}\}\}$ and $\mathbf{R}_{\mathbf{H}} = \mathbb{E}\{\mathbf{H}^{\text{H}}\mathbf{H}\}$.

- **Task 1:** Let $\mathbf{J} = \mathbb{E}\{||\mathbf{H} - \gamma\hat{\mathbf{H}}_{\text{LS}}||_F^2\}$. Find the optimal value of γ to minimize the channel estimation error.
- **Task 2:** Let $\hat{\mathbf{H}}_{\text{MMSE}} = \mathbf{Y}\mathbf{F}$. Determine the optimal $\mathbf{F}$ that minimizes $\mathbf{J} = \mathbb{E}\{||\mathbf{H} - \hat{\mathbf{H}}_{\text{MMSE}}||_F^2\}$. What is the optimal choice for $\mathbf{X}$?

Babak Hassibi

Babak Hassibi is a prominent electrical engineer and academic, celebrated for his contributions to wireless communications, information theory, and signal processing. He is the Mose and Lillian S. Bohn Professor of Electrical Engineering and Computing and Mathematical Sciences at Caltech, where he has also served in key leadership roles, including Chair of Electrical Engineering.

Hassibi earned his B.S. in Electrical Engineering from the University of Tehran and his M.S. and Ph.D. from Stanford University under Thomas Kailath. His groundbreaking work in MIMO systems, particularly in channel estimation methods like LS and MMSE, has revolutionized wireless communications. He is also recognized for advancing filtering, estimation, and adaptive signal processing. His work has earned prestigious honors, including the Presidential Early Career Award for Scientists and Engineers (PECASE) and the Packard Fellowship.

Read more in Wikipedia.
Reference: [1, 2].

Babak Hassibi
Born: 1967

References

[1] T. Kailath, A. H. Sayed, and B. Hassibi, *Linear estimation*. Prentice Hall, 2000.

[2] F. Oggier and B. Hassibi, "The secrecy capacity of the MIMO wiretap channel," *IEEE Transactions on Information Theory*, vol. 57, no. 8, pp. 4961–4972, 2011.